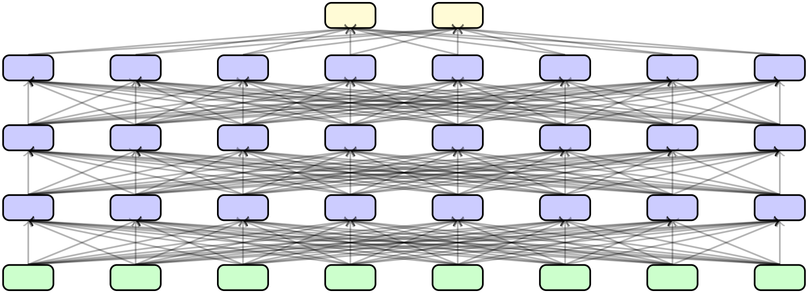
Tree-Structured Algorithms as Causal Abstractions of Neural Networks

Atticus Geiger

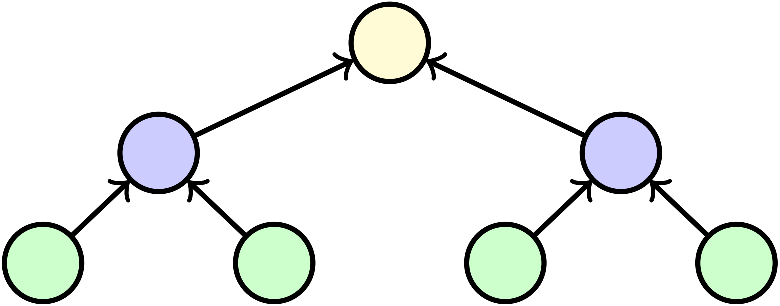
Stanford Linguistics and the Stanford NLP Group

Introduction

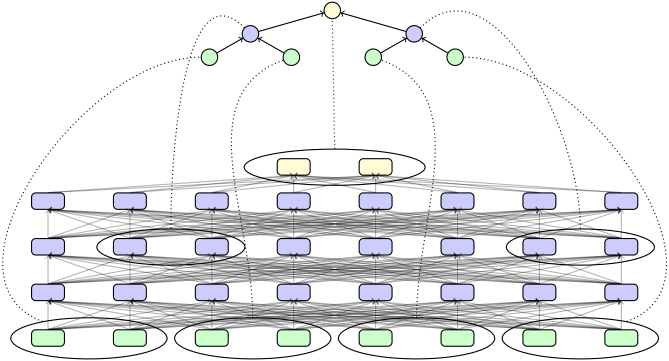
Deep Learning Model



Tree Structured Algorithm

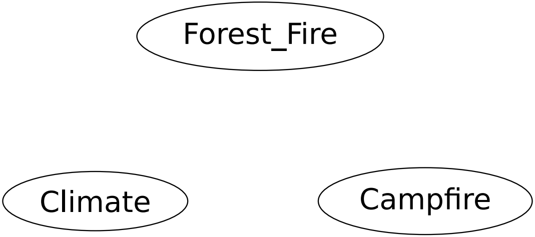


Constructive Causal Abstraction



Acyclic Causal Models

Dynamics with an Intervention



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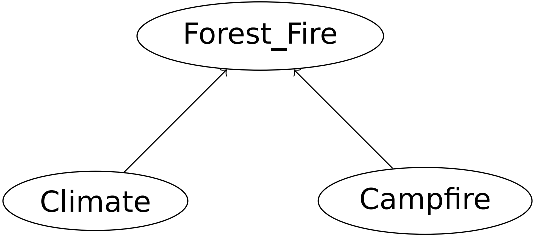
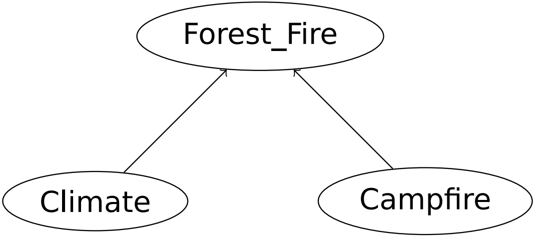


Dynamics with an Intervention

$$f_{\text{Forest\_Fire}}(\text{dry\_climate}, \text{campfire}) = \text{dry\_climate} \wedge \text{camp fire}$$



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$f_{\text{Campfire}} = \mathit{True}$

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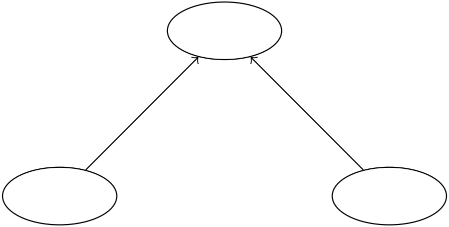

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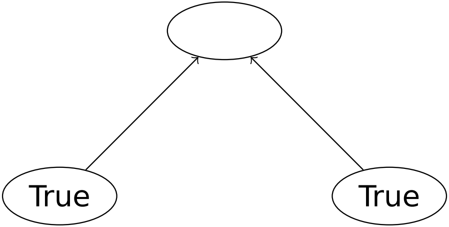

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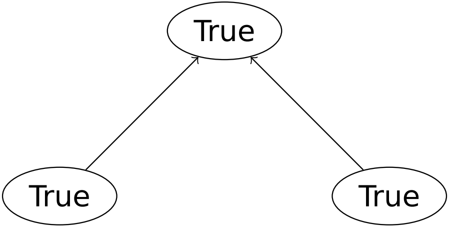

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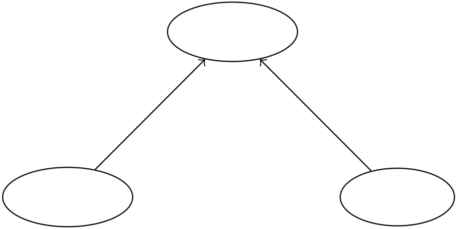

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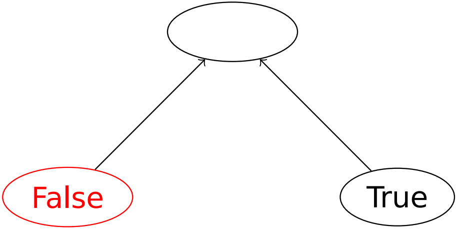

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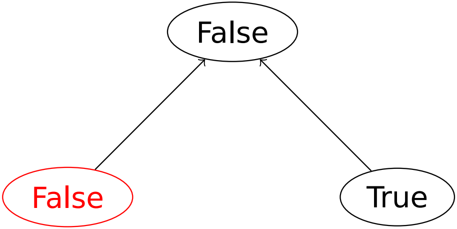

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Hierarchical Equality Task

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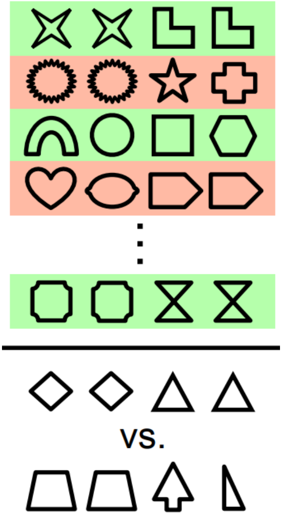


Algorithms as Acyclic Causal Models

Tree-Structured Algorithm

$\mathbf{Id_1}(\,\cdot\,,\,\cdot\,) =$

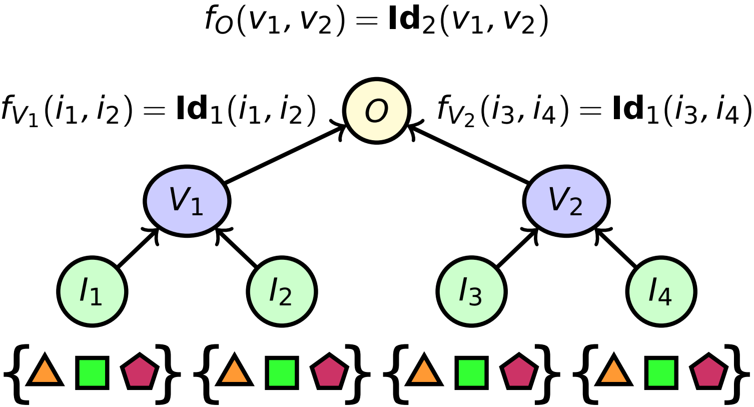
	△	🟩	⬟
△	T	F	F
🟩	F	T	F
⬟	F	F	T

$\mathbf{Id_2}(\,\cdot\,,\,\cdot\,) =$

	T	F
T	T	F
F	F	T

```
function EqualityTask($shape1, shape2, shape3, shape4$)
    $same1 \leftarrow \mathbf{Id_1}(shape1, shape2)$
    $same2 \leftarrow \mathbf{Id_1}(shape3, shape4)$
    $same3 \leftarrow \mathbf{Id_2}(same1, same2)$
    return $same3$
```

Causal Model of Algorithm



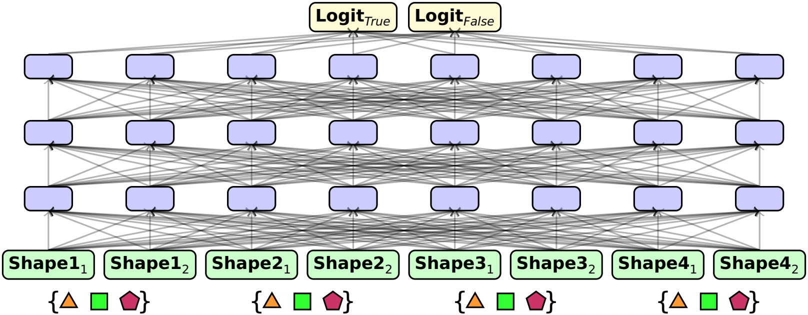
$$f_O(v_1, v_2) = \mathbf{Id}_2(v_1, v_2)$$

$$f_{V_1}(i_1, i_2) = \mathbf{Id}_1(i_1, i_2)$$

$$f_{V_2}(i_3, i_4) = \mathbf{Id}_1(i_3, i_4)$$

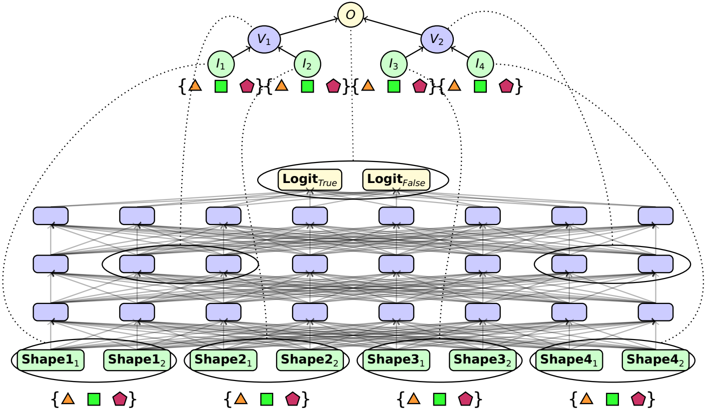
Deep Learning Models as Acyclic Causal Models

Deep Learning Models



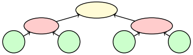
Constructive Causal Abstraction

Alignment



O
V_1
V_2
I_1
I_2
I_3
I_4
{▲ ■ ⬟}{▲ ■ ⬟}{▲ ■ ⬟}{▲ ■ ⬟}
Logit$_{True}$
Logit$_{False}$
Shape1$_1$
Shape1$_2$
Shape2$_1$
Shape2$_2$
Shape3$_1$
Shape3$_2$
Shape4$_1$
Shape4$_2$
{▲ ■ ⬟}
{▲ ■ ⬟}
{▲ ■ ⬟}
{▲ ■ ⬟}

Interchange Interventions



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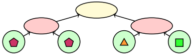

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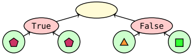

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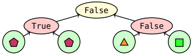

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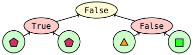

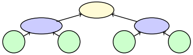

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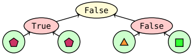

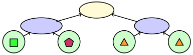

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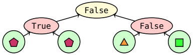

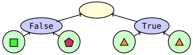

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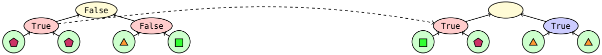

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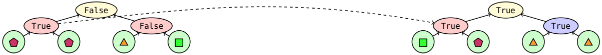

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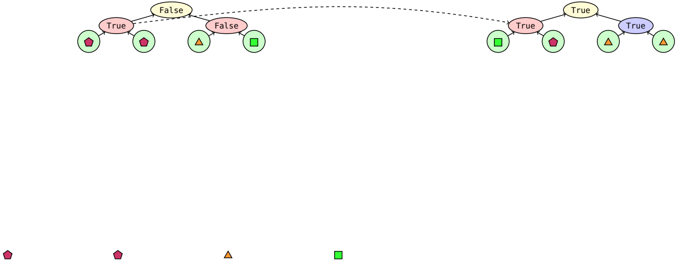

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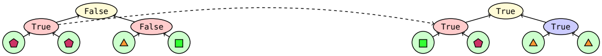

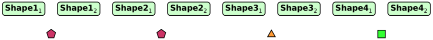

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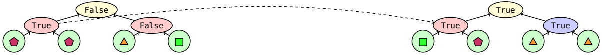

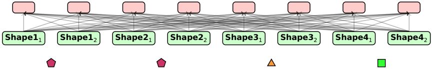

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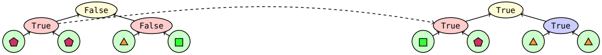

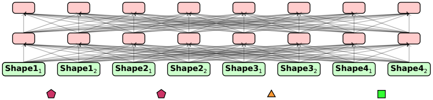

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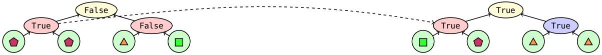

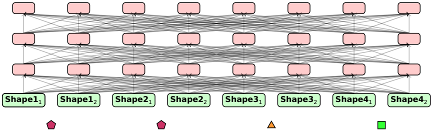

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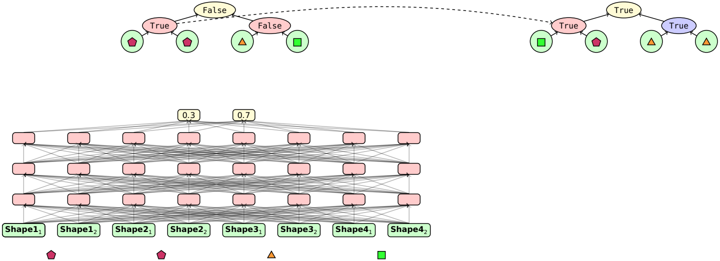

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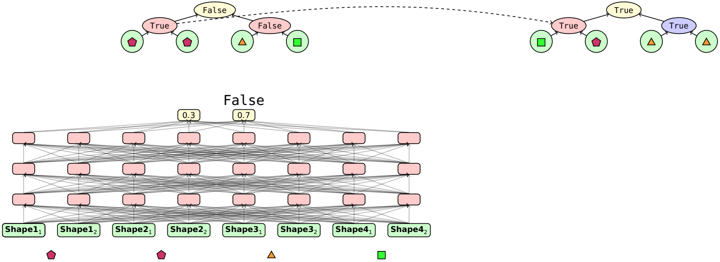

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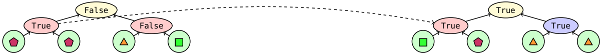

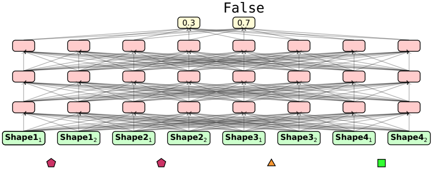

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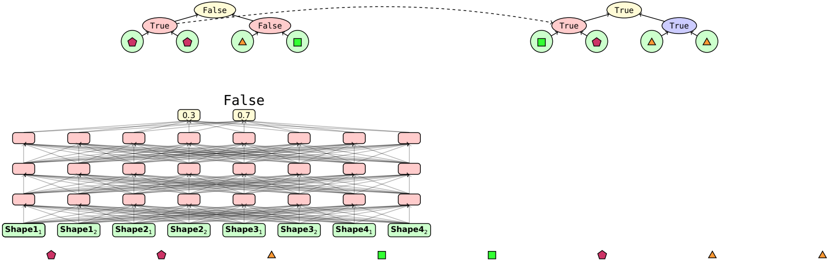

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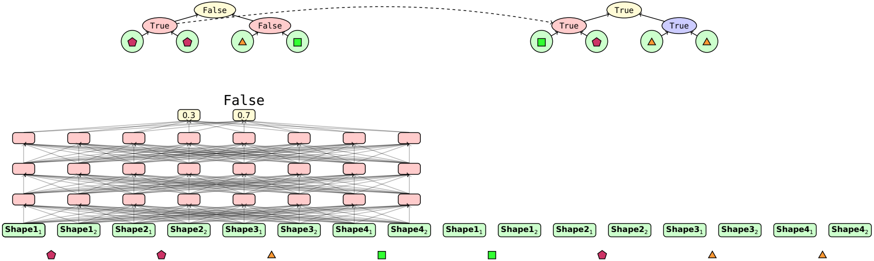

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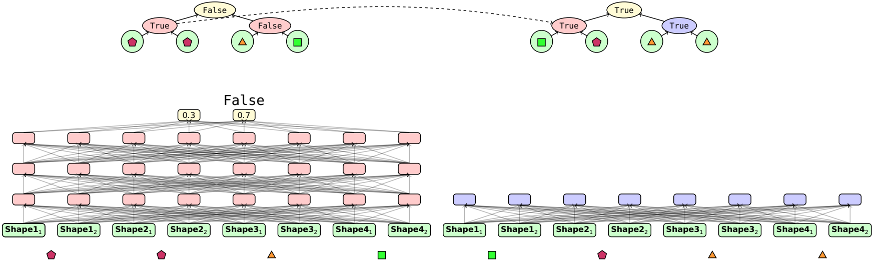

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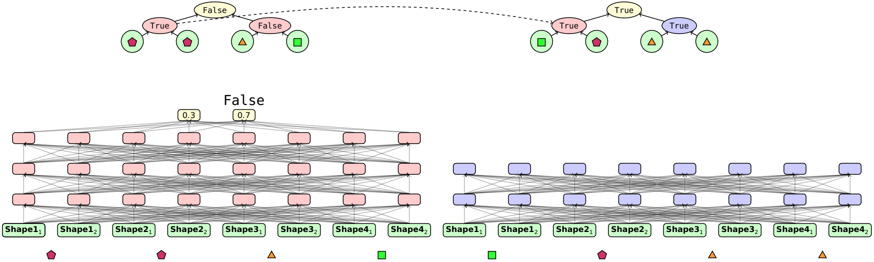

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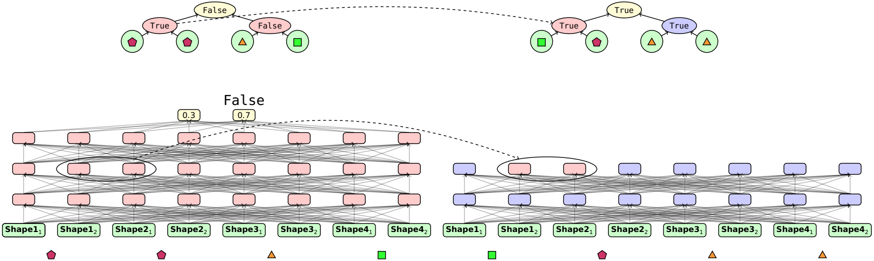

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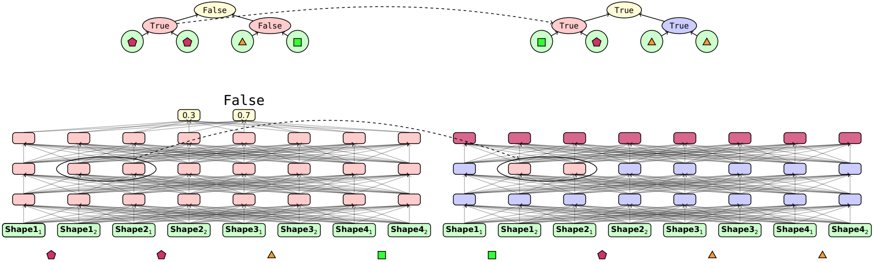

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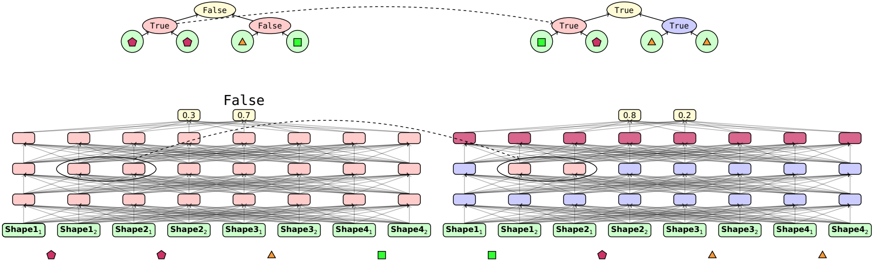

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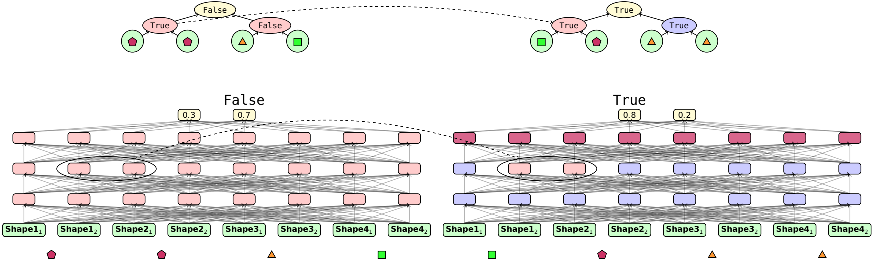
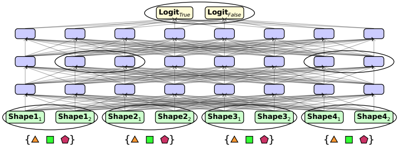


Marginalize



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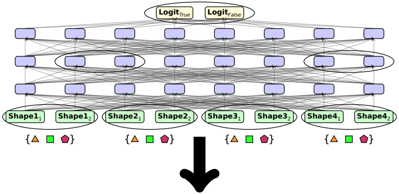

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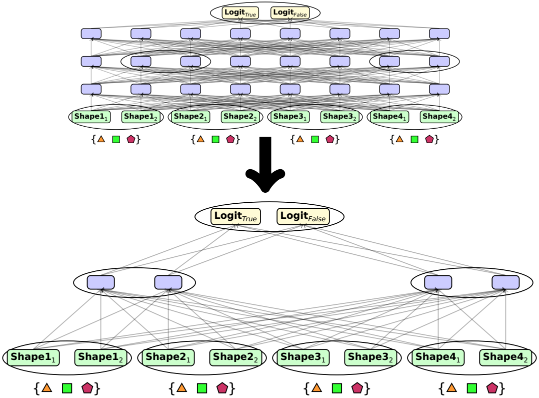
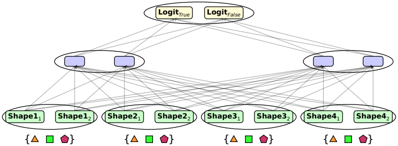
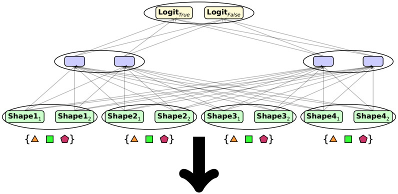


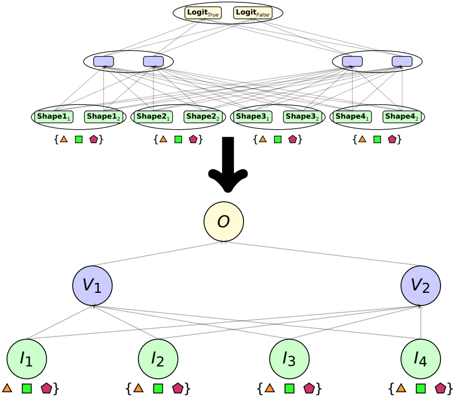
Variable Merge



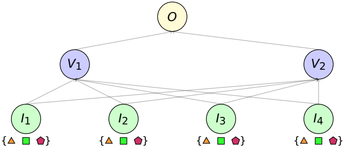
Variable Merge



Variable Merge



Value Change



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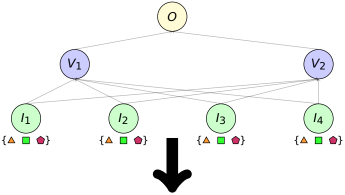

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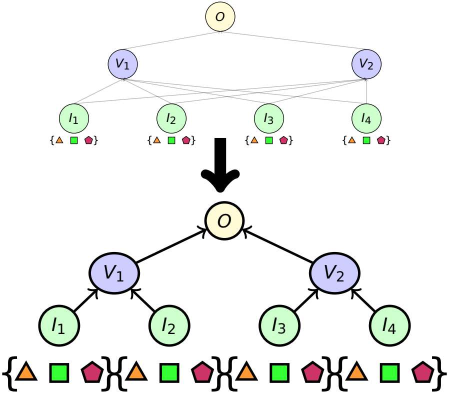
O
V1
V2
I1
I2
I3
I4
O
V1
V2
I1
I2
I3
I4